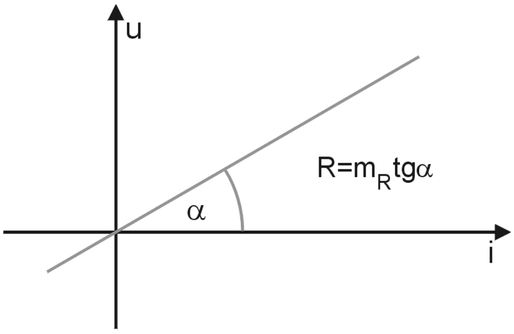
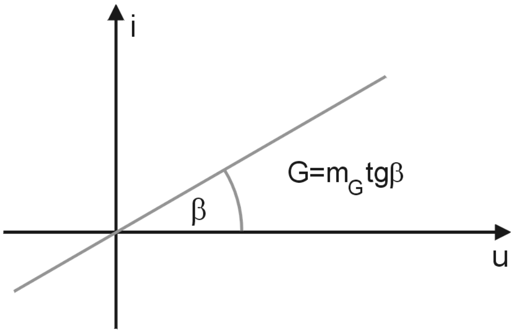
PRAWO OHMA

$\mathbf{R = \frac{U}{I}\,[\Omega]}$

$\mathbf{i = \frac{1}{R}\cdot u = G\cdot u}$





$$\mathbf{p = i\cdot u = R\cdot i^2 = G\cdot u^2 \geq 0}$$

$$\mathbf{W_R = u(t_0,t) = \int_{t_0}^{t} p(\tau)dt = R\int_{t_0}^{t} i^2 dt = G\int_{t_0}^{t} u^2 dt}$$

Niemalejąca funkcja czasu, konduktancja pobiera energię z otoczenia

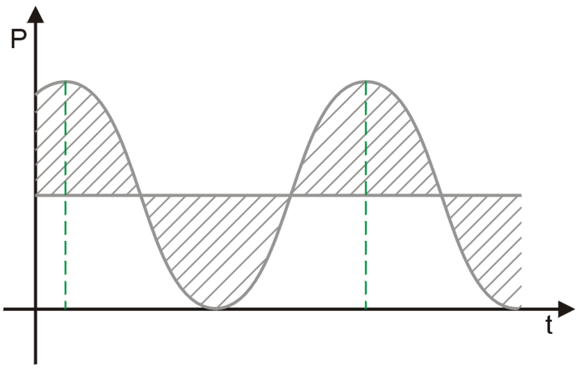
PRZYKŁAD:

1) Sygnały: (a) $\mathbf{i = I_0\ const}$ (b) $\mathbf{i = I_m \cos(\omega t+\varphi)}$ (c) $\mathbf{i = I_0\cdot e^{-2t}}$ $\mathbf{t \geq 0}$

(a) $\mathbf{p = R\cdot i^2 = R\cdot I_0^2 = const}$

(b) $\mathbf{p = R\cdot i^2 = R\cdot I_m^2 \cos(\omega t+\varphi) = \frac{1}{2}\, R\cdot I_m^2 [1+ \cos(2\omega t+2\varphi)]}$

$\mathbf{\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1}$



Zadanie domowe: $\mathbf{W_R(0,\infty)}$
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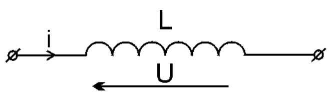
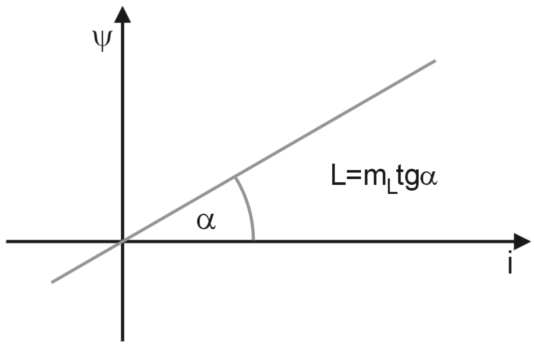
2.2.2 Indukcyjność liniowa



$\mathbf{\Psi = L \cdot i}$
1Wb = 1Hz·1A



Cewka indukcyjne ma zdolność gromadzenia i przechowywania energii pola magnetycznego

$$\omega(t)\int i\,d\Psi = L\int i\,di = \tfrac{1}{2}Li^2 \qquad d\Psi = L\,di$$

RÓWNANIA ZACISKOWE:

$\mathbf{u = L\frac{di}{dt}}$ - zależność jednoznaczna (z prawa Farraday'a)

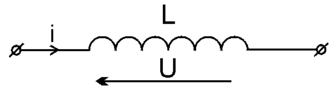
$$i = \tfrac{1}{L}\int u\,dt$$

$$i = \tfrac{1}{L}\int_{-\infty}^{t} u(\tau)d\tau = \tfrac{1}{L}\int_{-\infty}^{0} u(\tau)d\tau - \tfrac{1}{L}\int_{0}^{t} u(\tau)d\tau = i\left(0^-\right) + \tfrac{1}{L}\int_{0}^{t} u(\tau)d\tau$$

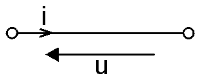
$$\omega(-\infty,0) = \tfrac{1}{2}Li\left(0^-\right)$$

PRZYKŁAD:

Sygnały: (a) $\mathbf{i = I_0\ const}$ (b) $\mathbf{i = I_m \cos(\omega t+\varphi)}$ (c) $\mathbf{i = I_0 \cdot e^{-2t}}$ $\mathbf{t \geq 0}$



(a) $\mathbf{u = L \frac{d}{dt}(I_0) = 0}$



w układzie prądu stałego w stanie ustalonym cewkę możemy zastąpić zwarciem, tak jak na rysunku obok

(b) $\mathbf{u = L \frac{d}{dt}[I_m(\cos\omega t+\varphi)] = LI_m \frac{d}{dt}\cdot\cos(\omega t+\varphi)dt = -\omega LI_m \sin(\omega t+\varphi) =}$

$\mathbf{= \omega LI_m \cos(\omega t+\varphi-90^\circ) = U_m \cos(\omega t+\Psi)}$

$\mathbf{U_m = \omega L \cdot I_m}$

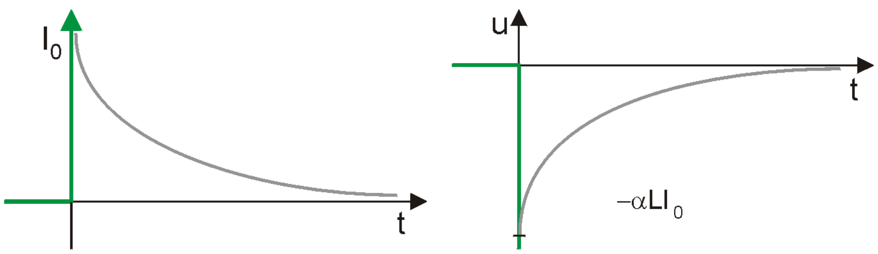
$\mathbf{1V = 1\Omega + 1A}$

U_m – amplituda napięcia

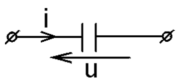
ψ - kąt fazowy o wartości $\varphi+90^{o}$

Napięcie na indukcyjności wyprzedza prąd o 90^{o}

(c) $\mathbf{u = L \frac{d}{dt} I_0 e^{-2t} = -\alpha LI_0 e^{-2t}}$



2.2.3 Pojemność liniowa



$\mathbf{q = C \cdot u}$

$\mathbf{C = \frac{q}{u}}$

$\mathbf{1F = 1\frac{C}{V}}$ $\mathbf{c \geq 0}$

Zależność między **u** oraz **i**

$\mathbf{i = \frac{dq}{dt} = C\frac{du}{dt}}$ $\mathbf{u = \frac{1}{C}\int i dt}$

energia gromadzona na pojemności

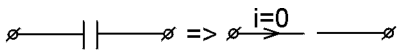
$\mathbf{w_c = \int u dq = \frac{1}{2}Cu^2}$ $\mathbf{dq = C\,du}$

$$\mathbf{u = \frac{1}{C}\int_{-\infty}^{0} i dt + \int_{0}^{t} i d\tau = U_c(0^-) + \frac{1}{C}\int_{0}^{t} i d\tau}$$

PRZYKŁAD:

(a) $\mathbf{u = U_0 = const}$

$$\mathbf{i = C\frac{du}{dt} = 0}$$



dla prądu stałego kondensator stanowi rozwarcie mimo, że na zaciskach jest napięcie – prąd nie płynie

(b) $\mathbf{u = U_m \cos(\omega t+\varphi)}$

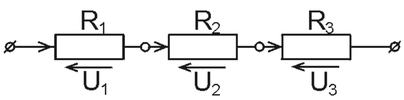
$$\mathbf{i = C\frac{du}{dt} = -\omega C U_m \sin(\omega t + \varphi) = \omega C U_m \cos(\omega t + \varphi + 90^\circ)}$$

prąd na pojemności wyprzedza napięcie o 90^o

$\mathbf{i = I_m \cos(\omega t+\psi)}$ $\qquad \mathbf{\psi = \varphi + 90^o}$

$\mathbf{I_m = \omega C U_m}$

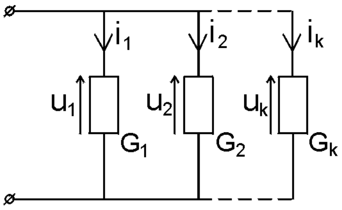
2.2.4 Zasady łączenia elementów

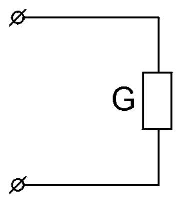


$$\mathbf{i_1 = i_2 = i_3 = \dots = i_k = i} \qquad \mathbf{U = \sum_{k=1}^{n} u_k}$$

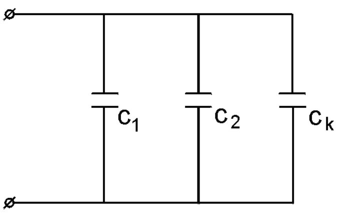
$$\mathbf{\frac{u}{i} = \frac{\sum_{k=1}^{n} u_k}{i} = \sum_{k=1}^{n}\frac{u_k}{i} = \sum_{k=1}^{n}\frac{u_k}{i_k} = \sum R_k}$$

$$\mathbf{R = \sum_{k=1}^{n} R_k} \qquad \mathbf{u_k = L_k \frac{di_k}{dt}} \qquad \mathbf{L = L_1 + L_2 + \dots = \sum_{k=1}^{n} L_k}$$



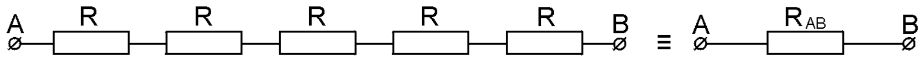


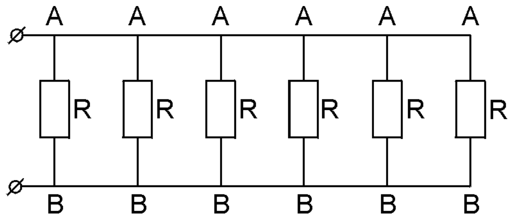
$$\mathbf{i = \sum_{k=1}^{n} i_k} \qquad \mathbf{U = U_1 = U_2 = \dots = U_k} \qquad \mathbf{G = \sum_{k=1}^{n} G_k} \qquad \mathbf{\frac{1}{R}\sum_{k=1}^{n}\frac{1}{R_k}}$$



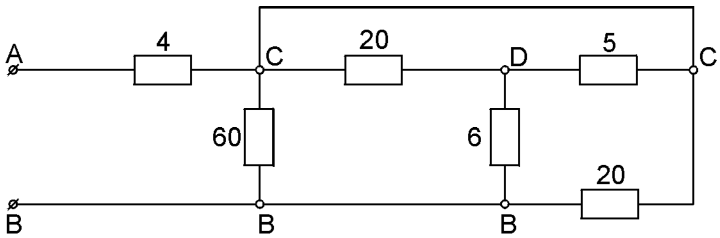
$$\mathbf{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1+R_2}{R_1 \cdot R_2}} \qquad \mathbf{C_1 + C_2 + \dots + C_k = C_z}$$

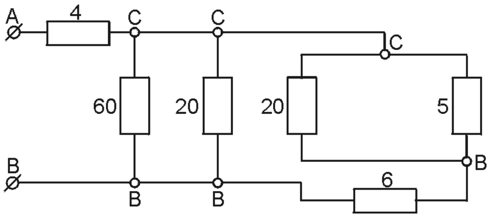
$$\mathbf{R = \frac{R_1 \cdot R_2}{R_1+R_2}}$$





$$R_{AB} = \frac{R}{5}$$



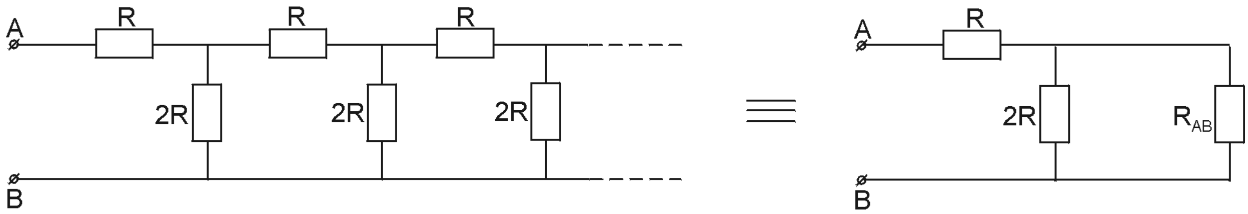


$$R_{CD} = \frac{100}{25} = 4$$

$$R_{CB} = 4 + 6 = 10$$

$$R_{CB} = \frac{1}{60} + \frac{1}{20} + \frac{1}{10} = 6$$

$$R_{AB} = 4 + 6 = 10$$



$$R_{AB} = R + \frac{2R \cdot R_{AB}}{2R + R_{AB}} = \frac{R(2R + R_{AB}) + 2R \cdot R_{AB}}{2R + R_{AB}}$$

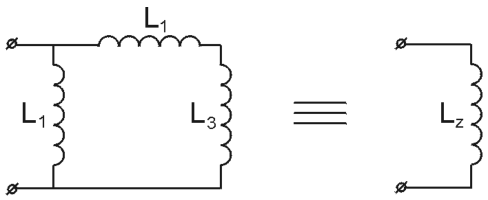
$$R_{AB}(2R + R_{AB}) = R(2R + R_{AB}) + 2R \cdot R_{AB}$$

$$2R \cdot R_{AB} - 2R^2 + 2R \cdot R_{AB} + R_{AB}^2 - R \cdot R_{AB} = 0$$

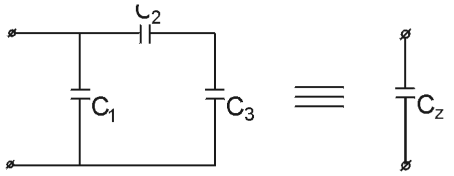
$$R_{AB}^2 - R \cdot R_{AB} - 2R^2 = 0$$

$$\Delta = R^2 - 4 \cdot (-2R^2) = R^2 + 8R^2 = 9R^2 \qquad \sqrt{\Delta} = 3R$$

$$R_{AB} = \frac{R+3R}{2} = 2R \qquad R>0 \qquad R_{AB} = \frac{R-3R}{R} = -R$$ - odrzucamy



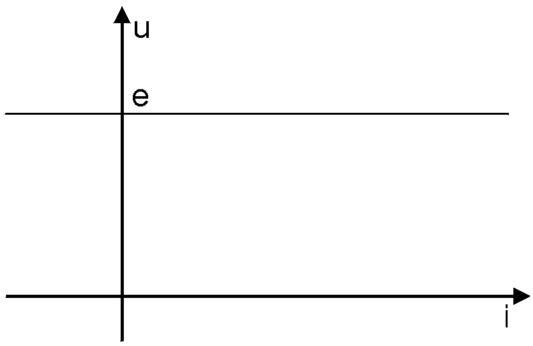
$$L_z = \frac{L_1 \cdot (L_2+L_3)}{L_1+L_2+L_3}$$

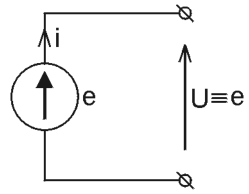


$$C_z = C_1 + \frac{C_2 \cdot C_3}{C_2+C_3} = \frac{C_1(C_2+C_3)}{C\ +C_3} + \frac{C_2 \cdot C_2}{C\ +C_3} = \frac{C_1 \cdot C_2+C_1 \cdot C_3+C_2 \cdot C_3}{C_2+C_3}$$

2.2.5 Źródła niezależne napięciowe

Idealne:

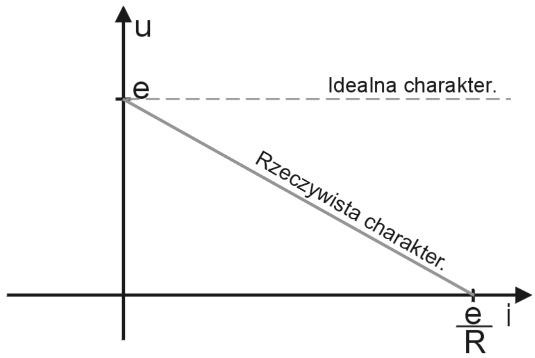


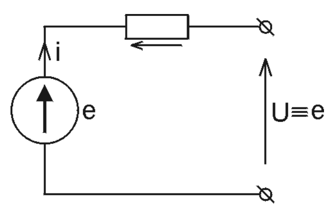


$\forall i \quad u \equiv e$

e – siła elektromotoryczna
$p = u \cdot i = e \cdot i \quad \geq 0$ moc dostarczona do obwodu ze źródła

Rzeczywiste:





$U = e - i \cdot R$
$i = 0 \Rightarrow \mathbf{u} = \mathbf{e}$
$u = 0 \Rightarrow \mathbf{i} = \frac{\mathbf{e}}{\mathbf{R}}$